Distributionally robust airline fleet assignment problem

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1 Summary

In this work we consider the airline fleet assignment problem. We experiment a robust solution where passenger demand is uncertain. To mitigate conservativeness of the classical robust optimization we consider a two-stage distributionally robust objective formulation (see [2] for concepts). The modeling characteristics of our proposal comprises our main original contributions with respect to the airline fleet management problem literature.

We assume that the demand uncertainty belongs to a known deterministic uncertainty set and this set is the support for the family of probability distributions associated with our random passenger demand parameter. We consider a data-driven approach by which this uncertainty set is constructed from available historical passenger demand data and using machine learning techniques.

Since fleet assignment decisions are made on daily basis, we optimize the worst case expected performance defined over an infinite number of probability distributions enclosed on a, so called, ambiguity set. The ambiguity set is constructed based on partial distributional information, namely, the support set defined above and moment statistics, also obtained from available historical data. We show how the choices of our ambiguity set lead to tractable reformulations of our problem.

We also propose a two-stage model. While all the fleet assignments decisions are first stage, the calculation of lost revenue (spill) is only done after realization of uncertainty.

We benchmark against current deterministic and robust fleet assignment formulations and verify solution performance results through simulation.

2 Problem definition

The fleet assignment problem consists of assigning aircraft fleet types to flight legs. This way seat capacity is assigned to each flight leg in a given airline schedule. A flight leg is the basic unit of aircraft operation with a takeoff from a departure airport and a landing at an arrival airport. The problem is constrained by the available number of aircraft per fleet. Fleets are also specific in the subset of legs they can operate and in the operating cost at which they operate them. Each fleet type provides a defined capacity, which implies that some legs are assigned a capacity that cannot accommodate all the passenger demand. This non-accommodated demand is said to be spilled. Passenger demands are defined at the itinerary level. An itinerary consists of a specific sequence of scheduled flight legs, in which the first leg originates from the origin airport at
a particular time and the final leg terminates at the final destination airport at a later time. Itineraries with different fare classes are defined within the airline schedule.

The itinerary-based representation of demand leads to a high granularity of demand, making it hard to predict. In [1], for example, the authors suggest different itinerary demand grouping strategies in order to minimize the effect of its volatility.

In this work we leverage the itinerary-based model defined in [1] but consider the random nature of passenger demand and use historical data to construct an ambiguity set of its probability distribution. By constructing the ambiguity set from historical data we are able to capture correlations between demands of different itineraries and thus mitigate the demand granularity effect.

Our objective function (1) minimizes the total cost of operations plus the worst case average cost related to spilled itinerary fare class demand. We assume the uncertainty of passenger demand vector $D$ is represented through a probability distribution $\mathbb{P}$ that belongs to an ambiguity set $\mathcal{D}$.

\[
\min \sum_{i \in L, k \in K} c_{k,i} f_{k,i} + \sup_{\mathbb{P} \in \mathcal{D}} \mathbb{E}_{\mathbb{P}}\{Q(f, D)\}
\]

where $c_{k,i}$ is the cost of operating leg $i$ with fleet type $k$, $f_{k,i}$ is a binary variable equal to 1 if fleet type $k$ is assigned to flight leg $i$ and 0 otherwise, and $Q(f, D)$ is given by:

\[
\min \sum_{p \in P} \text{fare}_p \, t_p(D)
\]

\[
\sum_{p \in P} \delta^p D_p - \sum_{p \in P} \delta^p t^p(D) \leq \sum_{k \in K} f_{k,i} \, \text{SEATS}_k \quad \forall i \in L
\]

\[
t_p(D) \leq D_p \quad \forall p \in P
\]

where $\text{fare}_p$ is the fare class for itinerary $p$, $t_p$ is the number of passengers requesting itinerary fare class $p$ and spilled by the model because of the capacity limit, $\delta^p$ is a binary flag equal to 1 if itinerary fare class includes flight leg $i$ and 0 otherwise, $D_p$ is the demand for itinerary fare class $p$ and $\text{SEATS}_k$ is the number of seats available on aircraft of fleet type $k$.

We define and use a moment-based ambiguity set, $\mathcal{D}$ as

\[
\mathcal{D} = \left\{ \mathbb{P} \in \mathcal{P}_0(\mathbb{R}^{\mid P\mid}) \left| \begin{array}{l}
\mathbb{E}_{\mathbb{P}}[g_i(D)] \leq \gamma_i \quad \forall i \in I \\
\mathbb{P}(D \in U) = 1
\end{array} \right. \right\}.
\]

where $\mathcal{P}_0(\mathbb{R}^{\mid P\mid})$ represents the set of all probability distributions in $\mathbb{R}^{\mid P\mid}$ and upper-bound parameters are defined as $\gamma \in \mathbb{R}^{\mid I\mid}$. We also define support set $U \in \mathbb{R}^{\mid P\mid}$ and functions $g_i \in \mathbb{R}^{\mid P\mid \times 1}$. We show how our choices of $U$ and $g_i$ lead to tractable reformulations of the problem.

References
