VRPTW with alternative paths on a road-network for perishable food

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1 Introduction

Nowadays, the distribution of perishable food in urban area is increasingly seen as a challenge for suppliers due to customer’s requirement and the characteristics of these products. Perishable food can be labeled as highly perishables, because their quality decay significantly fast over the time. Besides, customers generally require to be delivered during a specific time window, and a non respect of the latter may impose a penalty on the supplier.

For an effective and efficient distribution planning, suppliers should schedule their delivery trucks through solving a Vehicle Routing Problem with Time Window (VRPTW). Vehicle Routing Problem (VRP) have been widely studied in the literature, although most approaches are built on an assumption that each pair of nodes is linked by one best path, and hence, the problem can be addressed using a customer-based graph. Yet in real life, several attributes can be defined on each road segment (travel cost, distance, travel time, etc). Therefore each pair of nodes may be linked with a set of alternative paths, with compromises between the considering attributes. Consequently, representing the problem with a customer-based graph can discard good solutions and lead to bad optimization of the problem.

In VRPTW, if we consider that each road segment is defined by a traveling cost and time. The best path for the attribute cost, which is the cheapest, is unlike to be the fastest due to the traffic congestion on this segment for example. Since we are dealing with a problem subject to time window, decision maker may prefer an expensive road segment to avoid congestion and deliver product at time. This issue was addressed by two alternative approaches [1], the first consist of representing the road network with a multi-graph and the second solve the problem directly on a road-network graph. In this paper we adopt the second approach to solve the VRPTW considering alternative paths and we denote this problem version as VRPTW-P.

2 Problem formulation

The problem considered in this paper, can be defined as the routes construction for a set of vehicle, to meet customer’s demand in a specific time window. The overall objective is minimizing the total cost, composed of transportation costs, refrigeration costs and penalty costs already expressed on our previous work [2].
Our problem can be stated as follow: let \(G=(V,A)\) be the multi-graph, \(V\) is a set of \(n+2\) nodes where node 0 and \(n+1\) denotes depot. We have a set of \(K\) vehicles, each with a capacity \(Q\) to transport products from the depot to customers’ locations, \(C=(1, 2, \ldots, n)\) is the set of customers.  
\[
A = \bigcup_{(i,j)\in A} A_{(i,j)} 
\]
denotes the set of arcs where \(A_{(i,j)} = \{(i,j)^p, p=1, \ldots, |A_{(i,j)}|\}\) is the set of alternative paths linking node \(i\) to \(j\). We associate with each customer \(i\) a demand \(d_i\), a time window \([a_i, b_i]\) and a service time \(s_i\). Two attributes \(C^u_{(i,j)}\) and \(t^{sp}_{i,j}\) are associated to each arc \((i,j)^p\) respectively the travel cost and time to go from \(i\) to \(j\) through the path \(p\). We define the following decision variables:

\[
x^{(i,j)^p}_k: \text{ A binary variable equals to 1 if vehicle } k \text{ travels on arc } (i,j)^p \text{ and 0 otherwise}
\]

\[
t^i_k: \text{ The starting service time at customer } i \text{ if it is served by vehicle } k
\]

\[
y^i_k: \text{ is a binary variable equal to 1 if the vehicle } k \text{ serves for customer } i, \text{ 0 otherwise}
\]

The model is represented as follow:

\[
\begin{align*}
\text{Min} & \quad \sum_{k} \sum_{i \in V} \sum_{j \in V} \sum_{p=1}^{\left|A_{(i,j)}\right|} (C^u_{(i,j)^p} x^{(i,j)^p}_k d_i + C^c_{(i,j)^p} x^{(i,j)^p}_k t^{sp}_{i,j}) + \left(c \sum_{k} \sum_{i \in V} \sum_{j \in V} \sum_{p=1}^{\left|A_{(i,j)}\right|} x^{(i,j)^p}_k t^{sp}_{i,j} \right) \\
\text{Subject to:} & \quad \sum_{k} \sum_{j \in V} \sum_{p=1}^{\left|A_{(i,j)}\right|} x^{(i,j)^p}_k = 1 \quad i \in C \quad (2) \\
& \quad \sum_{i \in V} \sum_{p=1}^{\left|A_{(i,j)}\right|} x^{(i,j)^p}_k = 1 \quad k \in K \quad (3) \\
& \quad \sum_{i \in V} \sum_{p=1}^{\left|A_{(i,j)}\right|} x^{(i,j)^p}_k = 1 \quad k \in K \quad (4) \\
& \quad \sum_{i \in V} \sum_{j \in V} \sum_{p=1}^{\left|A_{(i,j)}\right|} d_j x^{(i,j)^p}_k \leq Q \quad k \in K \quad (5) \\
& \quad t^i_k + s_i + t^{sp}_{i,j} - M (1-x^{(i,j)^p}_k) \leq t^i_j \quad i,j \in V \quad k \in K \quad 1 \leq p \leq |A_{(i,j)}| \quad (7) \\
& \quad a_i \leq t^i_k \leq b_i \quad i \in V \quad k \in K \quad (8) \\
& \quad x^{(i,j)^p}_k \in \{0,1\} \quad i,j \in V \quad k \in K \quad 1 \leq p \leq |A_{(i,j)}| \quad (9) \\
& \quad \sum_{k} y^i_k \leq 1 \quad i \in C \quad (10) \\
& \quad t^i_k \geq 0 \quad i \in C \quad (11) \\
& \quad y^i_k \in \{0,1\} \quad i \in C \quad k \in K \quad (12)
\end{align*}
\]

3 Experimentation and perspectives

The model was solved in CPLEX and tested using small size instances. The next step in our work will be to develop optimization methods based on heuristics, to address instances of realistic size.

References
