Decentralized Spillover Algorithm for Capacitated Lot Sizing Problem

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1 Introduction and problem definition

In this paper, we study decentralized approaches to the multi-item, deterministic, single-level, capacitated lot sizing problem (CLSP). Here, single-level means that the end item is directly produced in one step as a single operation. Moreover, capacitated problem means there is a scarcity between the available limited production resources and the production demand.

Given a known deterministic forecast of the production demand of a set of items $I$ for a planning horizon $T$ made of $m$ time periods, the objective of the CLSP consists in determining the amount and the timing of the production of items (a production plan) in the planning horizon with a minimum cost, where the capacity restrictions constrain the production quantity in each period (see, e.g., [3, 6]).

Apart from production and manufacturing, CLSP formulation can be effectively used in many other scenarios like grid computing, energy management, healthcare, disaster management, and in transportation. Large amounts of generated data such as global spatio-temporal information of a myriad of agents, measurements of their states, and historical data are used both in the model building and model calibration processes.

The CLSP in this context requires an efficient and scalable solution approach due to the complexity of the model, the computational volume, and the amount of generated data. It is a very challenging and well-researched strongly NP-hard production-planning problem for which centralized and decentralized solution approaches exist (see, e.g., [1, 2]). Centralized (exact and heuristic) approaches assume only one actor in the decision making process while the decentralized ones, based on negotiation and auctions, consider the presence of multiple and competitive self-concerned agents. Moreover, centralized exact methods reach optimality but are not time-effective, while centralized heuristics do not have quality of solution guarantees, but are computationally fast. While centralized solution approaches are not adapted for parallel execution in high performance computing environments, the decentralized approaches do not provide quality of solution guaranties and are computationally expensive (see, e.g., [4, 5]). Therefore, a time-effective and optimal decentralized algorithm for the CLSP problem is still an open challenge.

2 Solution approach

In this paper, we develop a decentralized mathematical model and the heuristic solution approach for the CLSP and focus on key performance indicators related with the running time and the quality of solution.
Aiming at decision distribution, scalability, and low time complexity, we propose a decentralized heuristic algorithm that is based on the spillover effect. The latter relates to the impact of the production decision at one time period to the decisions in the neighboring periods, provoking, in the worst case, a chain reaction concerning the whole time horizon.

Inspired by the spillover effect, we model the CLSP by considering a multi-agent system paradigm with two kinds of agents: container and liquid agents. The time horizon made of $n$ time periods is modelled as $n$ container agents, each of which with a capacity equal to the overall production capacity at the shop floor at that time period. Each container is connected at the top with $m \cdot n$ channels (one per each item and period). Positive production demand $d^t_i$ of an item $i \in I$ at time period $t \in T$ is represented by a liquid agent rising from the bottom of container $t$. The volume of liquid $i \in I$ is equal to the value of the production demand $d^t_i$. The spillover effect occurs when the container capacity is lower than the volume of all liquids assigned to that container. Then, the volume of the liquid (a part of demand $d^t_i$) that is not accommodated in container $t$ propagates to the neighboring containers through the channels connected from above. The propagation occurs in the direction and order that follows the relative increase of the overall production costs related with demand $d^t_i$.

Each container is an agent with the following decision making process. Whenever a new request for accommodation of a liquid agent is received, the container orders all the received requests in a non-increasing order of their accumulated costs and assigns to it the liquids with the highest accumulated costs up to its capacity.

In the case of a too high demand, some or all volume of one or more liquid agents may remain outside of the $m$ containers. This volume (if any) represents unsatisfied demand whose overall cost is equal to or lower than the overall cost of any other liquid accommodated by the containers.

The spill-over algorithm runs asynchronously and optimizes the overall production costs in two dimensions: i) for each time period it finds the best assignment of limited resources to items based on the balance between the demand and the limited available resources and ii) for the given time horizon, it balances the usage of resources among individual time periods based on the spillover effect. Our hypothesis is that the spillover effect is an optimal strategy assuming that all the costs are positive. We prove this hypothesis by induction. Moreover, we demonstrate the functioning of the proposed spill-over algorithm in a simple manufacturing shop-floor example and compare it with the results obtained in CPLEX.

Références