Pricing-Allocation Bi-level Model in Combinatorial Auctions for Full Truckload Transportation Procurement

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1 Introduction

The auction-based Transportation Procurement Problem (TPP) involves solving the Bid Generation Problem (BGP) for carriers and the Winner Determination Problem (WDP) for shippers. Combinatorial auctions [1] allow a single bid of distinct lanes (i.e. a bundle), a lane is defined as a pick-up/delivery location pair. Combining distinct lanes into a single bid enables carriers to propose attractive prices, which will also lead to procurement cost reduction for shippers. In BGP, carriers place their bids (prices) for the most profitable bundles; in WDP, shippers allocate bundles to carriers based on their bids. To address this problem, we assume a market place representing shippers uses combinatorial auctions to allocate lanes to carriers. We study the Pricing-Allocation Problem (i.e. price-based BGP-WDP) under combinatorial auctions specially for full truckload transportation procurement. The BGP and WDP are two of the main auction phases in the combinatorial auctions. Even if they share strong relationship, it has not been investigated so far in the literature [3]. Our work is the first to merge BGP and WDP in a single bi-level formulation to model more accurate this interaction from a practical point of view.

2 Problem Formulation

2.1 Notation, definition and assumptions

Given \( V = \{1, \ldots, m\} \) the set of lanes and \( N = \{1, \ldots, n\} \) the set of carriers. A bundle \( S \) is a subset of lanes: \( S \subseteq V \). Let \( \mathcal{F} = \mathcal{F}_1 \cup \ldots \mathcal{F}_n \) be the set contains all the bundles, where \( \mathcal{F}_i \) is the bundle set proposed for carrier \( i \). In addition, we give \( \text{OperC}_i(\mathcal{F}_i) \), the operation cost if carrier \( i \) not covering any lane; \( \text{OperC}_i(S) \), the operation cost for carrier \( i \) to cover bundle \( S \); \( P_j \), the price the lane \( j \) is served by the spot market.

The decision variables of the problem consists: \( x_i(S) = 1 \) if bundle \( S \) is assigned to carrier \( i \), 0 otherwise; \( y_j = 1 \) if lane \( j \) is not covered by any carrier, 0 otherwise; \( z_i = 1 \) if carrier \( i \) is not covering any bundle, 0 otherwise; \( v_i(S) \) \( (\leq V(S) = \sum_{j \in S} P_j) \), the price proposed by carrier \( i \) for bundle \( S \).

2.2 Pricing-Allocation bi-level model

A pricing-allocation bi-level model is proposed to exploit the strong relationship between WDP and BGP. The carriers’ major objective in BGP is to take advantage of the inter-dependencies in
their transportation operations, and optimize their profit. The WDP determines an allocation of lanes given a set of carriers’ bids. The shipper’s objective is to minimize the total shipping cost, i.e. the sum of bidding prices of all bundles in the final allocation. The allocation determined by WDP is a response based on the carriers’ bids. This lead us to the following bi-level problem called the Pricing-Allocation bi-level model:

\[
\begin{align*}
\max_v & \sum_{i \in N} \sum_{S \in F_i} (v_i(S) - \text{OperC}_i(S)) x_i(S) - z_i \text{OperC}_i(F_i) \\
\text{s.t.} & \sum_{S \in F_i} x_i(S) + z_i = 1 \quad \forall i \in N,
\end{align*}
\]

(1) and (2) represent the BGP and WDP, respectively.

### 2.3 1-1 Pricing-Allocation and primal-dual reformulation

To simplify the problem, we consider a special case where bundles consist in individual lanes, and the number of bundles is identical to the number of lanes and to the number of carriers \( m = n \). Moreover, we assume that all carriers have the same bundle pool \( F_i = F = \{\{1\}, \{2\}, \ldots, \{m\}\}, \forall i \in N \). In such a case, the lower level turns to a linear assignment problem, where each carrier covers at most one bundle, i.e. one lane. Additional variables are associated with dummy carriers and dummy lanes, that is, if one carrier covers no lane, he gets a dummy lane; if one lane is not covered by any carrier, it is assigned to spot market dummy carrier. The 1-1 Pricing-Allocation Problem can be solved exactly by a primal-dual reformulation of the lower level.

### 2.4 Multi-leader single-follower bi-level model

Observe that, for both cases presented above, carriers are cooperating with each other to achieve the highest total profit, which does not capture the fact that carriers could adjust their prices to get more profit for their own. To model the competition among carriers at the upper level, multi-leader single-follower bi-level model is proposed. The carrier interaction at the upper level is modelled as a Nash Equilibrium which can be approximated by using the best response dynamics [2].

### 3 Conclusions

We describe a bi-level Model in combinatorial auctions to express the interaction between the BGP and the WDP, the problem aims to identify an optimal solution in which the carriers’ decisions are taken considering the shipper reaction. We are able to solve the 1-1 Pricing-Allocation exactly by replacing the lower level problem by its primal-dual optimality conditions; and Nash Equilibrium can be approached for the multi-leader single-follower bi-level model.

### References

