An Hypergraph Model for the Rolling Stock Rotation Planning and Train Selection

Mohamed Benkirane¹,²,³, François Clautiaux¹,³, Boris Detienne¹,³, Jean Damay²

¹ Université de Bordeaux, France, UMR CNRS 5251 IMB
mohamed.benkirane@u-bordeaux.fr
² Sncf Mobility, France
³ RealOpt Inria Bordeaux Sud-Ouest, France

Keywords: hypergraph, time-space flow model, rolling stock rotation, train selection

1 Introduction

The subject of this work is an integrated optimization approach for timetabling and rolling stock rotation planning in the context of passenger railway traffic. Our approach is based on an hypergraph based integer programming model, which can handle trains composed by multiple heterogeneous self-powered railcars. The method aims at producing a timetable and solving the rolling stock problem given a set of possible passenger trips, a service requirement and a fleet of self-powered railcars. The produced timetable optimizes the production cost and especially the use of railcars. To solve our optimization problem, we build a network-flow model in an hypergraph. We use these models to handle effectively constraints related to coupling and decoupling railcars. To reduce the size of the generated model, we use an hypergraph aggregation and disaggregation technique combined with a preprocessing based on reduced-cost filtering. We present some results based on several French regional railway traffic case studies to show that our methods scales to real-life problems.

2 Problem description

The railway production planning process combines heterogeneous resources and is usually decomposed into different sequential sub-problems, beginning by line planning, timetabling, rolling stock rotations and crew scheduling. Our approach aims at solving the timetable and rolling stock problems in an integrated manner.

The infrastructure resource allows to produce trips from a specific geographical point to another specific point at precise times. We call these trips trains. Given a line planning and service requirement constraints, the problem is to produce a timetable for a set of trains and the objective is to minimize the cost of the railcars used. An originality of our approach is to integrate train selection within rolling stock optimization.

The solution produced has to respect several constraints. Two trains covered by the same railcars must respect a minimum waiting duration, and the rolling stock planning has to be cyclic. The timetable produced is also subject to passenger demand constraints characterized by marketing issues. The frequency distribution of the trips and the number of seats available have to meet the demand requirements, which are expressed as follows. We define a set of train patterns ("missions"), with specific stopping points between origin and destination. To each mission, and for a set of time intervals, a demand is expressed as a desired frequency and a minimum number of seats available in this interval.
3 Solving approach

We model our problem as a minimum cost flow problem in a time/space/configuration hypergraph network. Flow models are largely used in the literature and can represent a flow of passengers, vehicles or agents.

Network-flow models have already been proposed for similar problems [1, 2]. The authors of [1] use a flow in an hypergraph model to solve the vehicle rotation problem. The proposed hypergraph is based on timetabled trips. The nodes correspond to a possible assembling of railcars to cover a trip and the hyperarcs correspond to possible reassembling. The authors of [2] use a time-space directed multigraph to solve the timetabling problem. The nodes of the graph represent a possible departure/arrival from/to a specific station and the arcs represent a possible trip between two different stations, or a waiting time.

We propose a new modeling approach based on a directed hypergraph. Our network is defined by a pair $(V, A)$ where $V$ is the set of nodes and $A$ is the set of hyperarcs. Each hyperarc $a \in A$ is defined by a pair of multisets $(T, H)$, $T$ is the multiset tail and $H$ is the multiset head. The idea behind using hyperarcs comes from representing constraints related to train configurations (ordered set of railcars), coupling and decoupling operations. Additionally, as path based constraints (maintenance constraints) are not taken into account in our problem, we aggregate in our hypergraph some nodes and we merge and split flow units. We also show that by using some hyperarc structures we combine several rail operations within the same hyperarc definition.

Binary variables corresponding to the use of hyperarcs in our hypergraph combined to some capacitated hyperarcs lead to the definition of our Mixed Integer Program. The output solution corresponds to a subset of arcs of the hypergraph network; this set is translated to an optimal timetable of trips where each path represents a succession of trains covered by the same railcar unit (the rolling stock planning).

A direct application of a general-purpose solver does not address real-life instances. To obtain an efficient method, we propose a reduced-cost fixing method based on a surrogate relaxation of the flow constraints. This relaxation allows to produce a relaxed model with a similar structure, but significantly less variables and constraints. We are able to produce both lower and upper bounds for the initial problem. These bounds allow to apply implicitly reduced-cost fixing techniques to the initial model. The obtained reduced version of the initial model is then solved by an MIP solver.

Our algorithms are tested on several French regional railway traffic case studies; we present some results and show the speed-up resolution brought by the aggregation techniques.

References
